Eur Phys J C **33**, s01, s404–s406 (2004)
Digital Object Identifier (DOI) 10.1140/epjcd/s2003-03-408-6

EPJ C direct
electronic only

Order-α_s^2 corrections to semi-inclusive DIS

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Received: 9 August 2003 / Accepted: 18 August 2003 /
Published Online: 19 September 2003 – © Springer-Verlag / Società Italiana di Fisica 2003

Abstract. We analyze the order-α_s^2 gluon initiated QCD corrections to semi-inclusive deep inelastic scattering. We focus in the most singular pieces of these corrections and discuss the prescription of overlapping singularities in more than one variable and their factorization.

1 Introduction

In recent years there has been an increasing interest in semi-inclusive deep inelastic scattering (SIDIS), driven both by crucial breakthroughs in the QCD description of these processes [1,2,3] and also by the incipient availability of data related to them [4].

Although QCD corrections to SIDIS are well known at LO [2,3], until recently [5] no computations had been done up to NLO accuracy, nor assessments of how relevant the non homogeneous scale dependence, induced by fracture functions in order to describe target fragmentation processes, might be. In LO, non homogeneous evolution effects are restricted to a relatively small kinematic region. This suggested to neglect these effects in many phenomenological analyses of polarized SIDIS, leading baryon production, and diffractive DIS [4].

In NLO the above mentioned kinematical restrictions are no longer present, which in principle may lead to important corrections. From a theoretical point of view, the computation of the SIDIS NLO corrections, and specifically the explicit check of factorization of collinear singularities involve also some subtleties which need close attention. At variance with the totally inclusive case, for the computation of the SIDIS NLO corrections it is necessary to keep additional variables unintegrated. This leads to entangled singularities in more than one variable which requires a detailed analysis of the singularity structure characteristic of the process and special prescription techniques+[5].

[a] Partially supported by CONICET, ANPCyT, UBACYT, and Fundación Antorchas.

2 Kinematics

The cross section for a one-particle inclusive process in which a lepton scatters off a nucleon and a hadron is tagged in the final state can be written as [2]

$$\begin{aligned}\frac{d\sigma}{dx_B\,dy\,dv_h\,dw_h} =& \\ &\sum_{i,j=q,\bar{q},g} \int_{x_B}^1 \frac{du}{u} \int_{v_h}^1 \frac{dv_j}{v_j} \int_0^1 dw\, f_{i/P}\left(\frac{x_B}{u}\right) D_{h/j}\left(\frac{v_h}{v_j}\right) \\ &\times \frac{d\hat{\sigma}_{ij}}{dx_B\,dy\,dv_j\,dw_j}\,\delta(w_h - w_j) \\ &+ \sum_i \int_{\frac{x_B}{1-(1-x_B)v_h}}^1 \frac{du}{u}\, M_{i,h/P}\left(\frac{x_B}{u}, (1-x_B)v_h\right) \\ &\times (1-x_B)\,\frac{d\hat{\sigma}_i}{dx_B\,dy}\,\delta(1-w_h)\,, \end{aligned} \qquad (1)$$

where in addition to the usual DIS variables x_B and y, we introduce energy and angular variables

$$v_h = \frac{E_h}{E_0(1-x_B)} \qquad w_h = \frac{1-\cos\theta_h}{2}. \qquad (2)$$

E_h and E_0 are the energies of the final state hadron and of the incoming nucleon in the $\mathbf{P}+\mathbf{q}=0$ frame, respectively. θ_h is the angle between the momenta of the hadron and the virtual photon in the same frame. The variable u is related to the fraction of momentum of the incoming parton ξ by $\xi = x_B/u$, while v_j and w_j are the partonic analogs of v_h and w_h.

The first term in (1), contain the partonic cross section which develop forward collinear singularities ($w_h = 1$) that can not be factorized in the usual partonic densities and fragmentation functions $f_{i/P}$ and $D_{h/j}$, respectively. This divergences are factorized into fracture functions $M_{i,h/P}$ and lead to their non homogeneous scale dependence.

$$\frac{\partial\, M_{i,h/P}(\xi,\zeta,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{\xi}{1-\zeta}}^{1}\frac{du}{u}\; P_{i\leftarrow j}(u)\, M_{j,h/P}\left(\frac{\xi}{u},\zeta,Q^2\right) + \frac{\alpha_s(Q^2)}{2\pi}\,\frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}}\frac{du}{u}\int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}}\frac{dv}{v}\, P_{ki\leftarrow j}(u,v) \times f_{j/P}\left(\frac{\xi}{u},Q^2\right) D_{h/k}\left(\frac{\zeta}{\xi\, v},Q^2\right) \quad (3)$$

The first order corrections to the one-particle inclusive cross section can be found in [2].

3 $\mathcal{O}(\alpha_s^2)$ corrections

At $\mathcal{O}(\alpha_s^2)$, the integration over the spectator partons [7] leads to a rich variety of singularities in the (u,v,w) space, regulated by the parameter ϵ. As it is standard in this kind of calculations, these singularities should be *prescribed* in order to get a series expansion in powers of ϵ suitable for making explicit their cancellation. These cancellations are performed by coupling constant renormalization for the UV singularities, by cancellations between virtual and real contributions for the soft ones, and by renormalization of parton densities, fragmentation and fracture functions in the collinear case. However, in the one-particle inclusive case, the structure of the singularities is much more complex than in the inclusive case, mixing the three variables and consequently the standard prescription

$$(1-u)^{-1+\epsilon} \equiv \frac{1}{\epsilon}\,\delta(1-u) + \left(\frac{1}{1-u}\right)_{+u[0,\underline{1}]} + \mathcal{O}(\epsilon)\,, \quad (4)$$

is no longer adequate. Indeed, it leads to double counting of the double poles and ill defined terms in the expansion. In Fig. 1 we show the curves along which the singularities in the regions $\mathrm{B0} = \{u \in [x_B, x_u],\ v \in [v_h, a],\ w \in [0,1]\}$ and $\mathrm{B1} = \{u \in [x_B, x_u],\ v \in [a,1],\ w \in [0, w_r]\}$ with $x_u = x_B/(x_B+(1-x_B)v_h)$ and $w_r = (1-v)(1-u)x_B/v(u-x_B)$, appear in the v-w plane after the angular integration is performed.

The prescription of overlapping divergences in these regions can be done using the modified prescription [5]

$$(1-w)^{-1+\epsilon_1}(w_r-w)^{-1+\epsilon_2} \xrightarrow{\mathrm{B0}} \frac{\Gamma(1+\epsilon_1)\,\Gamma(1-\epsilon_1-\epsilon_2)}{\epsilon_1\,(\epsilon_1+\epsilon_2)\,\Gamma(1-\epsilon_2)}\,\delta(1-w)\delta(a-v) \times (a-z)^{\epsilon_1+\epsilon_2}\,(a\,(1-a))^{1-\epsilon_1-\epsilon_2} + \frac{1}{\epsilon_1}\,\delta(1-w)\left((a-v)^{-1+\epsilon_1+\epsilon_2}\right)_{+v[z,\underline{a}]} \times\, (v\,(1-a))^{1-\epsilon_1-\epsilon_2}\; w_r^{-\epsilon_1}\;{}_2F_1\left[\epsilon_1,\epsilon_1+\epsilon_2,1+\epsilon_1;\frac{1}{w_r}\right] + \left((1-w)^{-1+\epsilon_1}(w_r-w)^{-1+\epsilon_2}\right)_{+w[0,\underline{1}]} \quad (5)$$

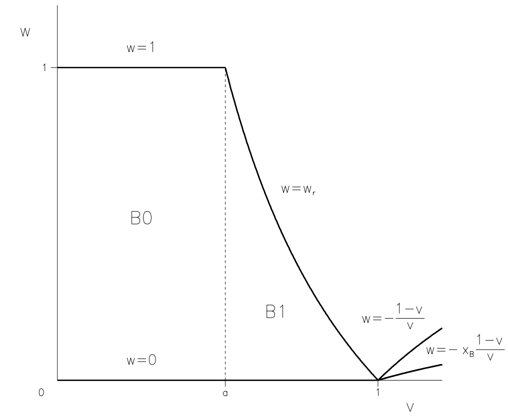

Fig. 1. Position of the singularities in the v-w plane for $x_B \le u \le x_u$. The bold lines represent the curves where the hadronic tensor becomes singular

and with a similar recipe for $\mathrm{B1} = \{u \in [x_B, x_u],\ v \in [a,1],\ w \in [0, w_r]\}$ [5].

4 Factorization

Once the renormalization of the coupling constant, and that for parton densities and fragmentation functions are accomplished, the remaining singularities occur in the region B0 and are proportional to $\delta(1-w)$, that is the forward direction, so they have to be factorized into renormalized fracture functions. Otherwise, factorization would be broken. The bare fracture functions can be written in terms of renormalized quantities as:

$$M_{i,h/P}(\xi,\zeta) = \frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}}\frac{du}{u}\int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}}\frac{dv}{v}\,\Delta_{ki\leftarrow j}(u,v,M_f) \times f^r_{j/P}\left(\frac{\xi}{u},M_f^2\right) D^r_{h/k}\left(\frac{\zeta}{\xi\, v},M_f^2\right) + \int_{\frac{\xi}{1-\zeta}}^{1}\frac{du}{u}\,\Delta_{i\leftarrow j}\,M^r_{j,h/P}\left(\frac{\xi}{u},\zeta,M_f^2\right) \quad (6)$$

where the factorization scale has been chosen to be the same for the three distributions. The functions $\Delta_{i\leftarrow j}$ and $\Delta_{ki\leftarrow j}$ are fixed in order to cancel all the remaining singularities in the cross section.

The non-homogeneous $\Delta_{ki\leftarrow j}$ for the case $j=g$, were obtained in [5]. Explicitly:

$$\Delta_{gg\leftarrow g}(u,v) = -\frac{\alpha_s}{2\pi}\,f_\Gamma\left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon/2}\frac{2}{\epsilon}\,\tilde{P}^{(0)}_{gg\leftarrow g}(u,v) \quad (7)$$

$$\Delta_{gq\leftarrow g}(u,v) = \left(\frac{\alpha_s}{2\pi}\right)^2 f_\Gamma^2\left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon}\left\{-\frac{1}{\epsilon}P^{(1)}_{gq\leftarrow g}(u,v) + \frac{2}{\epsilon^2}\left(\tilde{P}^{(0)}_{gq\leftarrow q}(u,v)\otimes P^{(0)}_{q\leftarrow g}(u) + \tilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\right.\right.$$

$$\otimes P^{(0)}_{g\leftarrow q}(v) + \tilde{P}^{(0)}_{gg\leftarrow g}(u,v) \otimes' P^{(0)}_{q\leftarrow g}(u)\Big)\Big\} \quad (8)$$

$$\Delta_{\bar{q}q\leftarrow g}(u,v) = -\frac{\alpha_s}{2\pi} f_\Gamma \left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon/2} \frac{2}{\epsilon} \tilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)$$
$$+ \left(\frac{\alpha_s}{2\pi}\right)^2 f_\Gamma^2 \left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon} \Big\{ \frac{2}{\epsilon^2}\Big(\tilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)$$
$$\otimes P^{(0)}_{g\leftarrow g}(u) + \tilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v) \otimes P^{(0)}_{q\leftarrow q}(v)$$
$$+ \tilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v) \otimes' P^{(0)}_{q\leftarrow q}(u)$$
$$+ \frac{1}{2}\beta_0 \tilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\Big) - \frac{1}{\epsilon} P^{(1)}_{\bar{q}q\leftarrow g}(u,v)\Big\} \quad (9)$$

where

$$f(u,v)\otimes g(u) = \int_u^{\frac{1}{1+v}} \frac{d\bar{u}}{\bar{u}} f(\bar{u},v) g\left(\frac{u}{\bar{u}}\right),$$
$$f(u,v)\otimes g(v) = \int_v^{\frac{1-u}{u}} \frac{d\bar{v}}{\bar{v}} f(u,\bar{v}) g\left(\frac{v}{\bar{v}}\right), \quad (10)$$
$$f(u,v)\otimes' g(u) = \int_u^{1-u\,v} \frac{d\bar{u}}{\bar{u}} \frac{u}{\bar{u}} f\left(\bar{u}, \frac{u}{\bar{u}}v\right) g\left(\frac{u}{\bar{u}}\right).$$

The homogeneous kernels $\Delta_{i\leftarrow j}$ are the same that appear in the inclusive case for parton densities and can be obtained from the corresponding transition functions in [6].

5 Scale dependence

Figure 2 compares (for different values of ξ and ζ) the relative size of the LO and NLO contributions to the non homogeneous term in the evolution equation (3) computed with standard sets of parton distributions [8] and fragmentation functions [9] for the case $i = q$ and $h = \pi^+$. The inset plots show the integral over Q^2 of this contributions. Notice that only those terms proportional to $f_{g/P}$ were taken into account in the $\mathcal{O}(\alpha_s^2)$ pieces. In [10] it was found that the LO non homogeneous contribution falls rapidly as ζ grows. This behavior is related to the shrinkage of the integration region and with the fall of fragmentation functions, $D_{h/i}(z)$, in the limit $z \to 1$.

This is also the case of the NLO contributions. At moderate and large values of ξ ($\xi \geq 0.1$) the $\mathcal{O}(\alpha_s^2)$ contributions are typically one order of magnitude smaller than the $\mathcal{O}(\alpha_s)$ ones so NLO and LO results differ only by a few percents. This can be traced back to the extra power of α_s and the small size of the integration region since the interval of the v integral in (3) shrinks to the point $(1-u)/u$ when $\xi \to 1-\zeta$. However, when ξ diminishes the integration region expands and NLO contributions grow considerably faster than the LO ones which are kinematically restricted to the curve $v = (1-u)/u$. The remarkable growth of the $\mathcal{O}(\alpha_s^2)$ terms makes these contributions even larger than the constrained $\mathcal{O}(\alpha_s)$ pieces at lower values of ξ and thus a priory non negligible in the evolution equations.

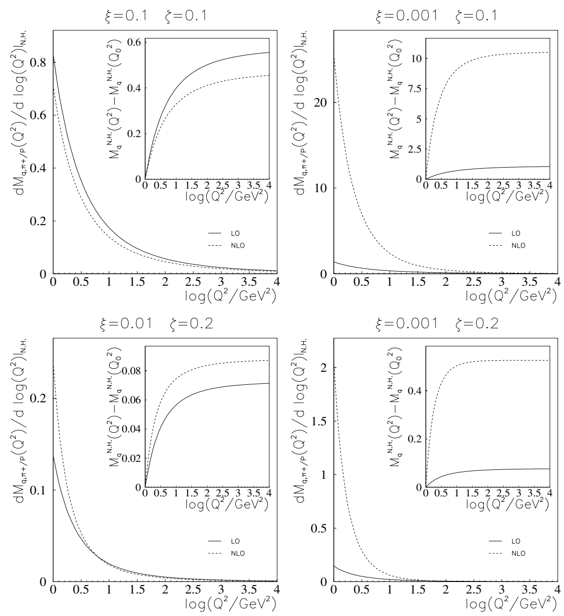

Fig. 2. Non homogeneous contributions to the derivative of M_q for different values of ξ and ζ. Inset plots show the integral over Q^2 of this contributions taking $M_q^{N.H.}(Q_0^2) = 0$ with $Q_0 = 1\,\text{GeV}$ as a reference

Of course, in order to assess the actual relevance of the NLO non homogeneous effects in the full evolution of fracture functions, one needs a realistic (based on actual data) estimate for the size and shape for these functions at a given scale, and compute the evolution taking into account all the appropriate kernels, but our present results suggest that non homogeneous NLO effects could be relevant.

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